

Compensating asynchrony effects in the calculation of financial correlations

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Abstract

We present a method to compensate statistical errors in the calculation of correlations on asynchronous time series. The method is based on the assumption of an underlying time series. We set up a model and apply it to financial data to examine the decrease of calculated correlations towards smaller return intervals (Epps effect). We show that this statistical effect is a major cause of the Epps effect. Hence, we are able to quantify and to compensate it using only trading prices and trading times.

Keywords: Financial correlations, Epps effect, Market emergence, Covariance estimation, Asynchronous time series

1. Introduction

The decrease of calculated correlations in financial data towards smaller return (or “sampling”-) intervals has been of interest since Epps discovered this phenomenon in 1979 [1]. Ever since, this behavior was found in data of different stock exchanges [2, 3, 4, 5] and foreign exchange markets [6, 7].

Many economists as well as physicists addressed this phenomenon, since a precise calculation of correlations is of major importance for the estimation of financial risk [8, 9, 10]. While the physicists’ approach is often to construct a model which offers an explanation for this phenomenon, the standard economy approach is to work on estimators with the aim to suppress the Epps effect. Recently, Hayashi and Yoshida introduced a cumulative estimator [11], only involving returns whose time intervals are overlapping. This estimator has been supplemented with different adjustments, such as bias compensation and *lead-lag* treatment [5, 12, 13]. A very similar approach on a completely different topic is the “discrete correlation function” in astrophysics which was introduced in 1988 by Edelson and Krolik [14]. Other approaches to estimate correlation coefficients involve *Previous-Tick-Estimators* [15, 16] or realized kernel functions [17].

An extensive study of microscopic causes leading to the Epps effect has been performed by Renò [18], while another work by Tóth et. al. introduce a model for the Epps effect which is based on the phenomenon of lagged correlations [19].

However, certainly miscellaneous mechanisms are contributing to the Epps effect. Thus our approach is different. First, we will introduce a simple model which offers an explanation for the statistical part of the Epps effect, based on a central assumption of an underlying time series. Secondly, based on that model, we will present an estimator, with which these effects can be compensated. Finally, we will quantify the impact of this phenomenon on the Epps effect in recent empirical data and show that it can be a major cause for the Epps effect, especially when looking at less frequently traded securities.

This paper is organized as followed: In section 2, we develop the model for correlations in asynchronous time series. Within the model, we observe a decay of correlations towards smaller return intervals, similar to the Epps effect. We then derive the method to compensate this phenomenon. In section 3, this method is applied to recent empirical data to estimate the impact of the observed effect on the Epps effect. We discuss the results in section 4.

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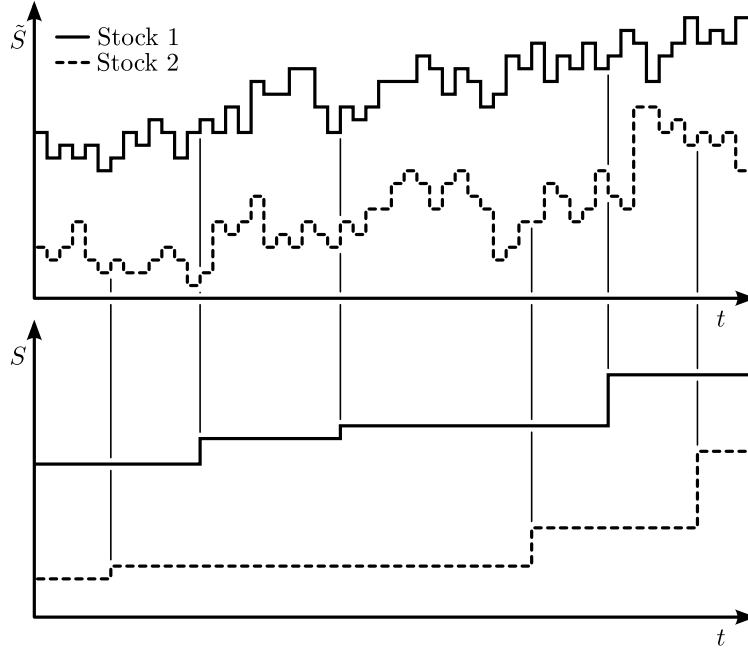


Figure 1: Illustration of the model for asynchronous trading times of two stocks. Shown above are the prices \tilde{S} on the underlying timescale. The “sampling” of these prices \tilde{S} to prices S on simulated trading times are shown below.

2. Statistical effects in asynchronous time series

In section 2.1, we set up our model and develop a compensation formalism for asynchrony effects in section 2.2.

2.1. Model

The central assumption of our model is the existence of an underlying non-lagged time series of prices. The assumption of a finer [19] or even continuous [11, 20, 18] underlying timescale is a common approach in the estimation of correlations. This approach is also intuitive, as most stocks are traded at several stock exchanges simultaneously.

To simulate asynchrony effects we generate an underlying correlated time series using the *Capital Asset Pricing Model* (CAPM) [21], which is also known as Noh’s model [22] in physics,

$$\tilde{r}^{(i)}(t) = \sqrt{c} \eta(t) + \sqrt{1-c} \varepsilon^{(i)}(t), \quad (1)$$

where $\tilde{r}^{(i)}$ stands for the relative price change, the so-called *return* of the i -th stock and c is the correlation coefficient. The random variables η and $\varepsilon^{(i)}$ are taken from a compound distribution as observed on market data by Gopikrishnan et. al. with power-law tails and a central Levy distribution (for details see Ref. [23]). We have chosen this approach to keep our model initially as simple as possible. We note, however, that return time series can also be autocorrelated. While first order autocorrelations are in this context insignificantly small [24], second order autocorrelations or “volatility clustering” represent a strong characteristic of return time series and led to the development of autoregressive models, such as GARCH [25, 26]. For this reason we also test our compensation in a more realistic setup against a GARCH(1,1) generated time series of underlying returns, given by

$$\tilde{r}^{(i)}(t) = \sigma^{(i)}(t) \left(\sqrt{c} \eta(t) + \sqrt{1-c} \varepsilon^{(i)}(t) \right) \quad (2)$$

with

$$\left(\sigma^{(i)}(t) \right)^2 = \alpha_0 + \alpha_1 \left(\tilde{r}^{(i)}(t-1) \right)^2 + \beta_1 \left(\sigma^{(i)}(t-1) \right)^2. \quad (3)$$

The initial parameters of the GARCH process have been chosen as $\alpha_0 = 2.4 \cdot 10^{-4}$, $\alpha_1 = 0.15$ and $\beta_1 = 0.84$.

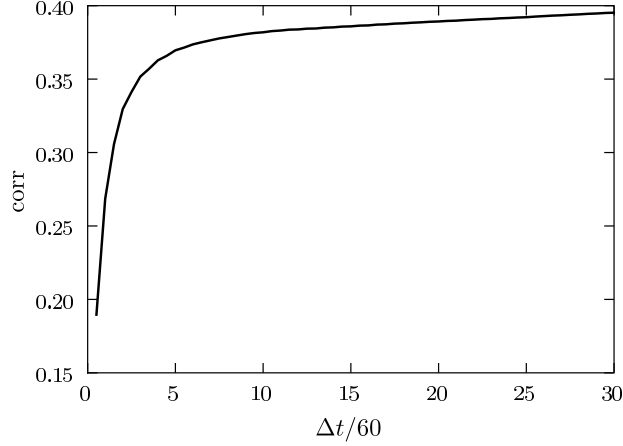


Figure 2: Scaling behavior of the correlation coefficient on a simulated asynchronous time series. The length of the underlying time series was set to $7.2 \cdot 10^6$ ($c = 0.4$, $\mu^{(1)} = 15$ and $\mu^{(2)} = 25$). The correlation coefficient was calculated on return-intervals from 60 data points (corresponding to 1 minute) to 1800 data points (corresponding to 30 minutes).

Two return time series $\tilde{r}^{(1)}$ and $\tilde{r}^{(2)}$ are generated representing two correlated stocks. The lengths of these underlying time series are chosen as $7.2 \cdot 10^6$, $1.44 \cdot 10^6$ and $7.2 \cdot 10^5$ corresponding to a return interval $\Delta \tilde{t}$ on the underlying timescale of 1, 5 and 10 seconds during 1 trading year.

Using these returns and an arbitrary starting price, the underlying price series $\tilde{S}^{(1)}$ and $\tilde{S}^{(2)}$ are calculated implying a geometric Brownian motion with zero drift and a standard deviation of 10^{-3} per time step. To model the asynchronous trade processes, these prices are sampled independently using exponentially distributed waiting times with average values typical for the stock market (see Fig. 1). In the following example, we choose the average waiting times as $\mu^{(1)} = 15$ and $\mu^{(2)} = 25$ (equivalent to seconds in this example), while the underlying time series were correlated with $c = 0.4$. On the resulting “macroscopic” time series, the return between two points in time (of the i -th stock) can be calculated as

$$r_{\Delta t}^{(i)}(t) = \frac{S^{(i)}(t + \Delta t) - S^{(i)}(t)}{S^{(i)}(t)}, \quad (4)$$

where $S^{(i)}(t)$ denotes the price at time t and Δt is the return interval. Between these return time series, we now calculate the correlation coefficient,

$$\text{corr}(r_{\Delta t}^{(i)}, r_{\Delta t}^{(j)}) = \frac{\langle r_{\Delta t}^{(i)} r_{\Delta t}^{(j)} \rangle - \langle r_{\Delta t}^{(i)} \rangle \langle r_{\Delta t}^{(j)} \rangle}{\sigma_{\Delta t}^{(i)} \sigma_{\Delta t}^{(j)}}, \quad (5)$$

where $\langle \dots \rangle$ denotes the mean value of a time series with length T and where σ refers to the standard deviation of the same time series. We note that we refer to the whole time series of returns $r_{\Delta t}^{(i)}$ when the argument (t) is omitted.

When calculating the correlation of returns of the sampled time series \tilde{S} using different return intervals Δt , the correlation coefficient scales down as shown in Fig 2. This behavior is very similar to the Epps effect in empirical data. It occurs only because of the asynchrony of the trading times. As this behavior is already observed in this simple setting, we are able to derive a method to compensate it, as the following demonstrates.

2.2. Compensation

The basic idea of this approach is the following: Due to the asynchrony, each term of the correlation coefficient can be divided into a part which contributes to the correlation and a part which is uncorrelated and therefore lowers the correlation coefficient.

According to the model assumption, the price change during Δt is based on price changes on an underlying “microscopic” timescale. Thus, the return can also be expressed as a sum of the underlying returns,

$$r^{(i)}(t) = \sum_{j=0}^{N_{\Delta t}^{(i)}(t)} \tilde{r}^{(i)}(\gamma^{(i)}(t) + j\Delta\tilde{t}) . \quad (6)$$

Here $\tilde{r}^{(i)}(t_i)$ is the return related to $S(t)$ on the underlying time scale of non-overlapping intervals $\Delta\tilde{t}$ (e.g. 1 second) given by

$$\tilde{r}^{(i)}(t + j\Delta\tilde{t}) = \frac{\tilde{S}(t + (j+1)\Delta\tilde{t}) - \tilde{S}(t + j\Delta\tilde{t})}{S(t)} . \quad (7)$$

The quantity $\gamma^{(i)}(t)$ in equation (6) represents the time of the last trade of the i -th stock at time t ,

$$\gamma^{(i)}(t) = \max(t_{\text{trade}}^{(i)}) \Big|_{t_{\text{trade}}^{(i)} \leq t} . \quad (8)$$

When calculating the return for the interval $[t, t + \Delta t]$ of two stocks, the actual price at t and $t + \Delta t$ is generally in the past, more precisely at $\gamma^{(1)}(t)$, $\gamma^{(2)}(t)$ and $\gamma^{(1)}(t + \Delta t)$, $\gamma^{(2)}(t + \Delta t)$. These trading times are distinct for each stock, therefore only a fraction of the underlying prices processed by the return is correlated. The number of terms $N_{\Delta t}^{(i)}$ of the sum in equation (6) is given by

$$N_{\Delta t}^{(i)}(t) = \frac{(\gamma^{(i)}(t + \Delta t) - \gamma^{(i)}(t))}{\Delta\tilde{t}} . \quad (9)$$

We normalize the returns to zero mean and unit variance and indicate them as g and \tilde{g} :

$$g_{\Delta t}^{(i)}(t) = \frac{r_{\Delta t}^{(i)}(t) - \langle r_{\Delta t}^{(i)} \rangle}{\sigma_{\Delta t}^{(i)}}, \quad \tilde{g}^{(i)}(t) = \frac{\tilde{r}^{(i)}(t) - \langle \tilde{r}^{(i)} \rangle}{\tilde{\sigma}^{(i)}} . \quad (10)$$

In this context, the relation of the returns on both time scales in equation (6) changes to

$$g_{\Delta t}^{(i)}(t) = \sqrt{\frac{\Delta\tilde{t}}{\Delta t}} \sum_{j=0}^{N_{\Delta t}^{(i)}(t)} \tilde{g}^{(i)}(\gamma^{(i)}(t) + j\Delta\tilde{t}) - \frac{\langle \tilde{r}^{(i)} \rangle \left(\frac{\Delta t}{\Delta\tilde{t}} - N_{\Delta t}^{(i)}(t) \right)}{\sigma_{\Delta t}^{(i)}} , \quad (11)$$

as worked out in appendix Appendix A. When using normalized returns, the correlation coefficient of two return time series $r_{\Delta t}^{(1)}$ and $r_{\Delta t}^{(2)}$ (see equation (5)) simplifies to

$$\text{corr}(r_{\Delta t}^{(1)}, r_{\Delta t}^{(2)}) = \text{corr}(g_{\Delta t}^{(1)}, g_{\Delta t}^{(2)}) = \frac{1}{T} \sum_{j=0}^T g_{\Delta t}^{(1)}(t_j) g_{\Delta t}^{(2)}(t_j) . \quad (12)$$

As the mean value over T of the second term from equation (11) is equal to zero, we obtain in terms of the underlying time series

$$\text{corr}(r_{\Delta t}^{(1)}, r_{\Delta t}^{(2)}) = \frac{1}{T} \sum_{j=0}^T \left(\sum_{k=0}^{N_{\Delta t}^{(1)}(t_j)} \tilde{g}^{(1)}(\gamma^{(1)}(t_j) + k\Delta\tilde{t}) \sum_{l=0}^{N_{\Delta t}^{(2)}(t_j)} \tilde{g}^{(2)}(\gamma^{(2)}(t_j) + l\Delta\tilde{t}) \right) \frac{\Delta\tilde{t}}{\Delta t} . \quad (13)$$

As illustrated in Fig. 3, only a subset of the underlying prices \tilde{S} of two prices S share an overlapping time-interval. Because of this “overlap” only a certain amount $\bar{N}_{\Delta t}(t)$ of the underlying returns is correlated, namely

$$\bar{N}_{\Delta t}(t) = \frac{\Delta t_0(t)}{\Delta\tilde{t}} \quad (14)$$

with $\Delta t_0(t)$ being the time interval of the actual overlap,

$$\Delta t_0(t) = \min(\gamma^{(1)}(t + \Delta t), \gamma^{(2)}(t + \Delta t)) - \max(\gamma^{(1)}(t), \gamma^{(2)}(t)) . \quad (15)$$

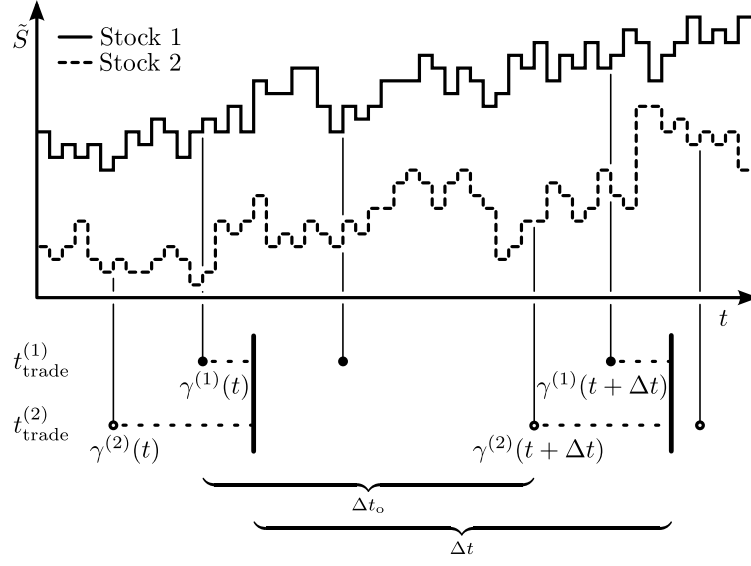


Figure 3: Illustration of the overlap Δt_o .

Each sum can be split up into $N_{\Delta t}^{(i)} - \bar{N}$ terms that are uncorrelated and \bar{N} that are correlated. Thus, equation (13) can be written as:

$$\begin{aligned} \text{corr}(r_{\Delta t}^{(1)}, r_{\Delta t}^{(2)}) &= \frac{1}{T} \sum_{j=0}^T \left(\left(\underbrace{\sum_{k=\bar{N}_{\Delta t}(t_j)+1}^{N_{\Delta t}^{(1)}(t_j)-\bar{N}_{\Delta t}(t_j)} \tilde{g}^{(1)}(t_k)}_{\text{async.}} + \underbrace{\sum_{\bar{k}=0}^{\bar{N}_{\Delta t}(t_j)} \tilde{g}^{(1)}(t_{\bar{k}})}_{\text{sync.}} \right) \right. \\ &\quad \times \left. \left(\underbrace{\sum_{l=\bar{N}_{\Delta t}(t_j)+1}^{N_{\Delta t}^{(2)}(t_j)-\bar{N}_{\Delta t}(t_j)} \tilde{g}^{(2)}(t_l)}_{\text{async.}} + \underbrace{\sum_{\bar{l}=0}^{\bar{N}_{\Delta t}(t_j)} \tilde{g}^{(2)}(t_{\bar{l}})}_{\text{sync.}} \right) \frac{\Delta \tilde{f}}{\Delta t} \right), \end{aligned} \quad (16)$$

where only the sums of synchronous returns are correlated among each other. In this notation, the underlying time series is indexed as $[\tilde{r}^{(i)}(t_0), \tilde{r}^{(i)}(t_1), \dots, \tilde{r}^{(i)}(t_{N_{\Delta t}^{(i)}})]$, where the returns from t_0 to $t_{\bar{N}_{\Delta t}}$ are corresponding to the overlap. When expanding the product, the non-correlated returns converge to zero due to the outer average

$$\begin{aligned} \text{corr}(r_{\Delta t}^{(1)}, r_{\Delta t}^{(2)}) &= \frac{1}{T} \sum_{j=0}^T \left(\underbrace{\sum_{k=0}^{\bar{N}_{\Delta t}(t_j)} \tilde{g}^{(1)}(t_k) \tilde{g}^{(2)}(t_k)}_{\bar{N}_{\Delta t}(t) \text{corr}_{t_j}(\tilde{r}_1, \tilde{r}_2)} + \underbrace{\dots}_0 \right) \frac{\Delta \tilde{f}}{\Delta t} \\ &= \frac{1}{T} \sum_{j=0}^T \text{corr}_{t_j}(\tilde{g}^{(1)}, \tilde{g}^{(2)}) \frac{\bar{N}_{\Delta t}(t) \Delta \tilde{f}}{\Delta t} \\ &= \frac{1}{T} \sum_{j=0}^T \text{corr}_{t_j}(\tilde{g}^{(1)}, \tilde{g}^{(2)}) \frac{\Delta t_o(t_j)}{\Delta t}, \end{aligned} \quad (17)$$

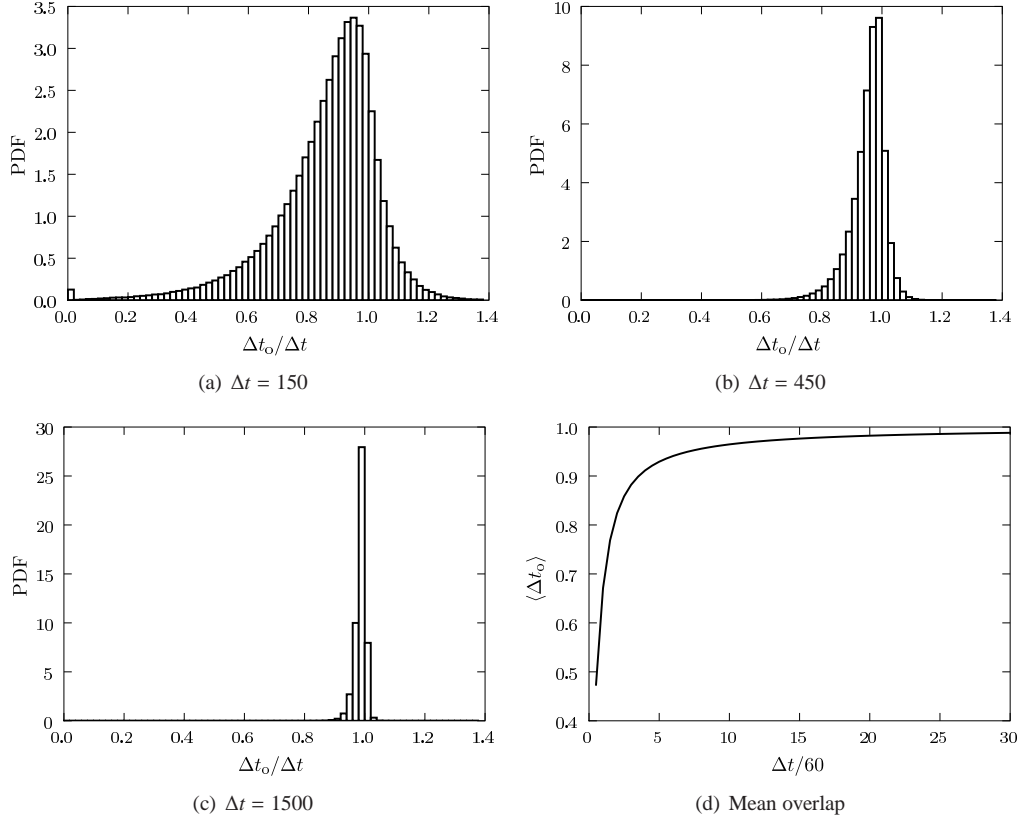


Figure 4: Distribution of the overlaps for simulated return intervals $\Delta t = 150$ (a), 450 (b) and 1500 (c) data points (corresponding to 2.5, 7.5 and 25 minutes). Towards larger return-intervals, the distribution sharpens, as well as its mean converges to 1 (d).

where corr_t represents the correlation of the underlying returns corresponding to the interval $[t, t + \Delta t]$.

$\Delta t_o(t)/\Delta t$ is the fractional overlap of the corresponding return interval. The fractional overlap does not depend on the actual timescale of the underlying time series. As equation (17) clearly shows, the correlation coefficient of the synchronous part of the return time series is multiplied by the fractional overlap. Hence, this effect can be compensated by

$$\text{corr}_{\text{corrected}}(r_{\Delta t}^{(1)}, r_{\Delta t}^{(2)}) = \frac{1}{T} \sum_{j=0}^T g_{\Delta t}^{(1)}(t_j) g_{\Delta t}^{(2)}(t_j) \frac{\Delta t}{\Delta t_o(t_j)}. \quad (18)$$

The dashed line in Fig. 5 represents the asynchrony-compensated correlation within our simulation. It turns out that there is a remaining effect that still causes a downscaling of the correlation coefficient for very small return intervals. This behavior occurs when the price of either of the stocks did not change during the return-interval and therefore the corresponding return equals zero. Of course, this event becomes more probable on smaller return intervals Δt . It corresponds to the small peak at $\Delta t_o = 0$ in Fig. 4(a). This remaining downscaling coincides with the cumulative estimator described by Hayashi and Yoshida [11]. It can also be expressed in the formalism used here. It reads

$$\text{corr}(r_{\Delta t}^{(1)}, r_{\Delta t}^{(2)}) \Big|_{(\gamma_1^{(1)}(t) \neq \gamma_1^{(1)}(t+\Delta t)) \wedge (\gamma_2^{(2)}(t) \neq \gamma_2^{(2)}(t+\Delta t))}. \quad (19)$$

Therefore, when combining both estimators, and thus only regarding returns with overlapping time intervals, the remaining scaling behavior for very small returns can be compensated as well.

As displayed in Fig. 4, the overlap can also be larger than the actual return interval, implying that terms with such overlaps are corrected downwards. Therefore the compensation can amplify a specific term of the correlation coefficient as well as it can attenuate it.

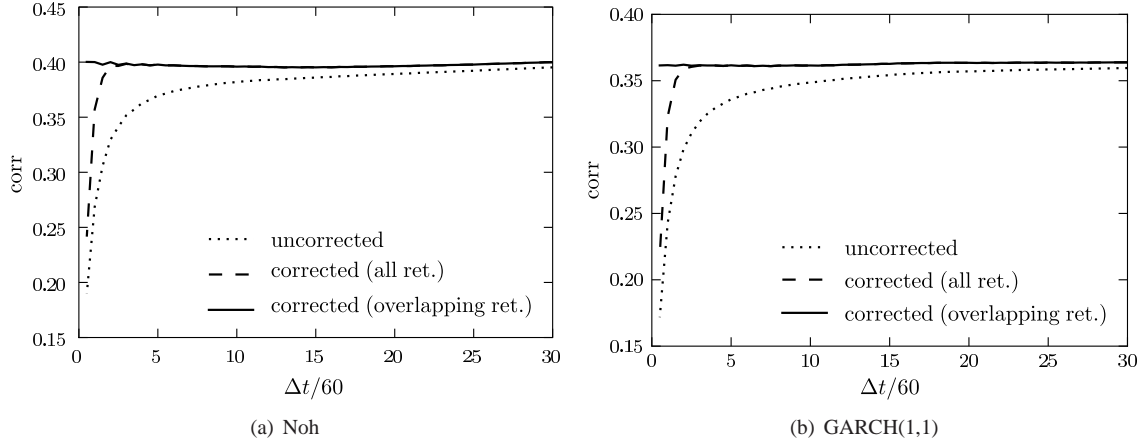


Figure 5: Compensation of asynchrony effects within the model with different approaches in the generation of the underlying time series. The dashed line represents the correlation coefficient, which is corrected by the overlap. The solid line regards in addition only returns, in which time intervals trades occurred.

3. Application to market data

Certainly, many aspects contribute to the Epps effect. Our present aim is to quantify the part, which is caused by the asynchrony of the time series.

It is difficult to isolate the Epps Effect on single stock pairs, as it can superimpose with other effects leading to other characteristics of the correlation coefficient than expected according to the Epps effect. A common approach on this topic is to pick the pairs of stocks, which show a distinctive Epps effect and focus the analysis to these pairs [3, 19, 12]. In the following, we would like to take a different approach:

We classify two ensembles of stock pairs. After compensating the asynchrony effect for each pair, we build the average for the ensemble. We also plot the error bars representing the double standard deviation 2σ . By this method, we can show the scope of the asynchrony model and identify regions, in which other effects dominate. All data was extracted from the NYSE's TAQ database for the year 2007 [27].

The first ensemble consists of stock pairs which provide the most stable correlation. Thereby we want to suppress those effects which are caused by a change in the correlation during the period in which the correlation coefficient is calculated. This ensemble represents ideal test conditions for the asynchrony compensation. To identify those stock pairs with a stable correlation, we calculate the correlation coefficient of 30 daily returns. After shifting this window in 1-day intervals through the year, we calculate the variance of the obtained correlation coefficients (var_{corr}). Then we identify the five stocks providing the smallest variance for each *Global Industry Classification System* (GICS) branch of the *Standard & Poor's* (S&P) 500 index. This results in an ensemble of 50 stocks as shown in table B.1, appendix Appendix B.

As the correlation structure of stocks can be non-stationary, we also evaluate the asynchrony compensation without the restriction to stable correlations. For this purpose, we select a second ensemble consisting of 5 stock pairs of each GICS branch of the S&P 500 index, whose daily returns are providing the strongest correlation during the year 2007. These stocks include highly non-stationary correlations as indicated in table B.2, appendix Appendix B (row " var_{corr} ").

Fig. 6 shows the ensemble average of the correlation coefficient and the asynchrony-compensated correlation coefficient for both ensembles in 2007 (250 trading days). Before averaging, the correlation coefficients for each stock have been normalized to the value at a return interval $\Delta t = 40$ minutes.

When looking at the whole ensemble we discover that the asynchrony has a pronounced impact on the Epps effect. The asynchrony effect seems to be the dominating cause for the Epps effect on return intervals down to approximately 10 minutes, where the remaining Epps effect is on average less than 3% of the correlation coefficient's saturation value at large return intervals. For smaller return intervals, other effects dominate, e.g. a lag between the time series of two

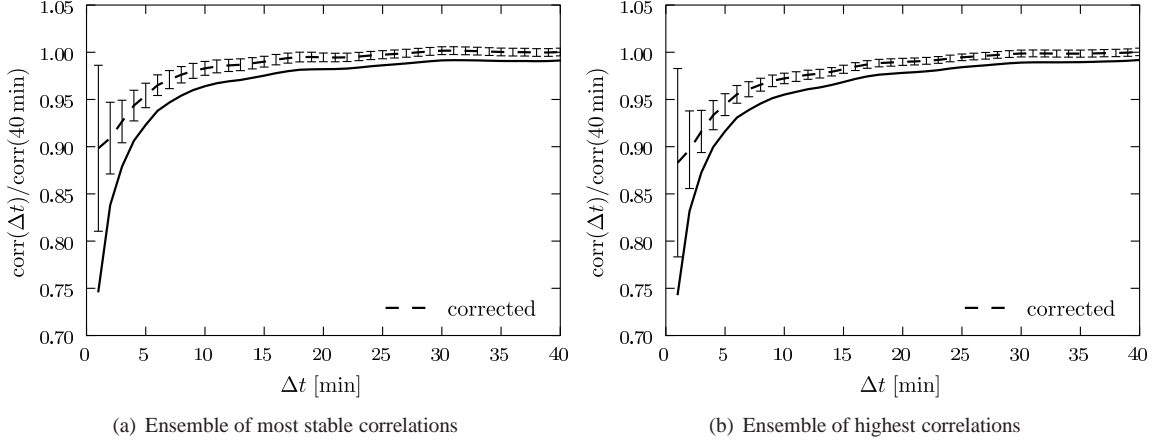


Figure 6: Asynchrony-compensated correlations of two ensembles. The data has been normalized to its value at 40 minutes. The error bars represent the double standard deviation.

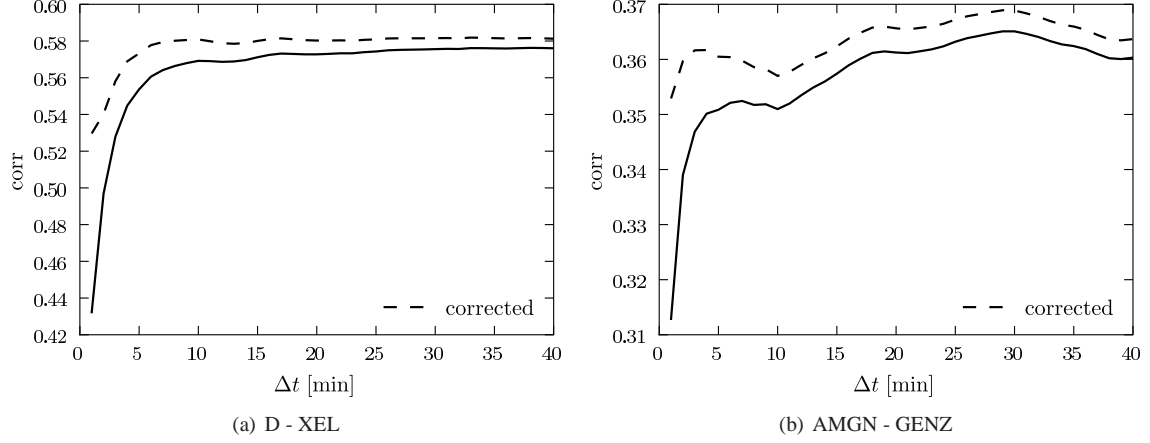


Figure 7: Asynchrony-compensated correlations between Dominion Resources, Inc. (D) - Xcel Energy Inc. (XEL) and Amgen Inc. (AMGN) - Genzyme Corp. (GENZ)

stocks, as recent study indicates [19].

However, the ensemble consists also of stocks which are very frequently traded, providing a very short average waiting time which results in a fractional overlap $\Delta t_o(t)/\Delta t$ close to unity. Evidently the presented compensation only has a small impact on the correlation estimation of these stocks, as they are so frequently traded that their time series can almost be described as continuous. Naturally the presented compensation works best for less frequently traded stocks, as they actually show an asynchronous behavior. Fig. 7 provides two examples of stock pairs for which the asynchrony of time series is a major effect. While Fig. 7(a) shows a “clean” Epps effect, Fig. 7(b) shows an Epps effect which is superimposed with other phenomena.

Of course, within the statistical ensemble stock pairs can be found that either do not show an Epps effect or that are so infrequently traded that the assumption of an underlying timeline seems to be unreasonable. Even though the assumption of an underlying time series is a common and intuitive approach on this topic, it may not be valid for very infrequently traded stocks.

When looking at single stock pairs, it turns out that the asynchrony-compensation works well, if a distinguished Epps effect is found. In case of adopting the presented method as a black box model without looking at the scaling

behavior of the correlation coefficient, we believe that a return interval of 5 minutes represents a good lower bound for the scope of this method.

4. Conclusion

We presented a model for the scaling behavior of financial correlations due to the asynchrony of the time series. This purely statistical effect can be compensated. Furthermore, we applied this compensation to market data under the assumption of an underlying time series with non-lagged correlations. We quantified the influence of the asynchrony on the overall decay of the correlation coefficient towards small return intervals, which is known as the Epps effect. The results clearly demonstrate that the asynchrony can have a huge impact on the Epps effect. It rather can be the dominating cause for less frequently traded stocks. The main advantage of our method is that no parameters or adjustments are necessary, since it is based on the trading times only.

In our empirical study, the asynchrony-compensation allowed us to recover the correlation coefficient for return intervals down to 10 minutes. At this return interval, the remaining Epps effect is on average less than 3% of the correlation coefficient's saturation value at large return intervals. We also demonstrated that the presented method holds for non-stationary correlated time series. The accurate calculation of correlations is of major importance for risk management. To keep the estimation error small, a long time series of returns is required. Yet at the same time, the time series should not reach too far into the past. The latter is important because the correlation structure can be highly dynamic, as the dramatic events of autumn 2008 prove. Applying our method to intraday data allows to choose smaller return intervals and hence provides improved statistical significance of the correlations for the same time horizon.

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Appendix A. Relation between $g_{\Delta t}^{(i)}$ and $\tilde{g}^{(i)}$

We defined the normalized returns as

$$g_{\Delta t}^{(i)}(t) = \frac{r_{\Delta t}^{(i)}(t) - \langle r_{\Delta t}^{(i)} \rangle}{\sqrt{\text{Var}(r_{\Delta t}^{(i)})}} \quad (\text{A.1})$$

$$\tilde{g}^{(i)}(t) = \frac{\tilde{r}^{(i)}(t) - \langle \tilde{r}^{(i)} \rangle}{\sqrt{\text{Var}(\tilde{r}^{(i)})}}, \quad (\text{A.2})$$

where $\text{Var}(\dots)$ refers to the variance of a time series. Inserting the return, expressed through the underlying time series,

$$r^{(i)}(t) = \sum_{j=0}^{N_{\Delta t}^{(i)}(t)} \tilde{r}^{(i)}(\gamma^{(i)}(t) + j\Delta\tilde{t}), \quad (\text{A.3})$$

in equation (A.1), results in

$$g_{\Delta t}^{(i)}(t) = \frac{\sum_{j=0}^{N_{\Delta t}^{(i)}(t)} \left(\tilde{r}^{(i)}(\gamma^{(i)}(t) + j\Delta\tilde{t}) - \langle r_{\Delta t}^{(i)} \rangle \right)}{\sqrt{\text{Var}(r_{\Delta t}^{(i)})}} \quad (\text{A.4})$$

$$= \frac{\sqrt{\text{Var}(\tilde{r}^{(i)})} \sum_{j=0}^{N_{\Delta t}^{(i)}(t)} \left(\tilde{g}^{(i)}(\gamma^{(i)}(t) + j\Delta\tilde{t}) - \langle r_{\Delta t}^{(i)} \rangle + N_{\Delta t}^{(i)}(t) \langle \tilde{r}^{(i)} \rangle \right)}{\sqrt{\text{Var}(r_{\Delta t}^{(i)})}}. \quad (\text{A.5})$$

In equation (A.5), (A.2) was used to express the underlying returns $\tilde{r}^{(i)}$. The mean values and variance are additive, which leads to

$$\langle r_{\Delta t}^{(i)} \rangle = \langle N_{\Delta t}^{(i)}(t) \rangle \langle \tilde{r}^{(i)} \rangle, \quad \text{Var}(r_{\Delta t}^{(i)}) = \langle N_{\Delta t}^{(i)}(t) \rangle \text{Var}(\tilde{r}^{(i)}) . \quad (\text{A.6})$$

Therefore, we obtain

$$g_{\Delta t}^{(i)}(t) = \frac{1}{\sqrt{\langle N_{\Delta t}^{(i)}(t) \rangle}} \sum_{j=0}^{N_{\Delta t}^{(i)}(t)} \tilde{g}^{(i)}(\gamma^{(i)}(t) + j\Delta\tilde{t}) - \frac{\langle \tilde{r}^{(i)} \rangle (\langle N_{\Delta t}^{(i)}(t) \rangle - N_{\Delta t}^{(i)}(t))}{\sqrt{\text{Var}(r_{\Delta t}^{(i)})}} . \quad (\text{A.7})$$

As the average time interval per return converges to Δt , the mean number of underlying price changes $\langle N_{\Delta t}^{(i)}(t) \rangle$ is given by $\Delta t / \Delta\tilde{t}$. Thus, we arrive at

$$g_{\Delta t}^{(i)}(t) = \sqrt{\frac{\Delta\tilde{t}}{\Delta t}} \sum_{j=0}^{N_{\Delta t}^{(i)}(t)} \tilde{g}^{(i)}(\gamma^{(i)}(t) + j\Delta\tilde{t}) - \frac{\langle \tilde{r}^{(i)} \rangle \left(\frac{\Delta t}{\Delta\tilde{t}} - N_{\Delta t}^{(i)}(t) \right)}{\sqrt{\text{Var}(r_{\Delta t}^{(i)})}} , \quad (\text{A.8})$$

which is equation (11).

Appendix B. Stock ensembles

Table B.1: Top 5 five stock pairs with the most stable correlation from each GICS branch of the S&P 500 index.

GICS Branch	Stock 1				Stock 2				corr	varcorr
	Symbol	Name	Stock Exchange	Volume	Symbol	Name	Stock Exchange	Volume		
Consumer Discretionary	AMZN	Amazon Corp.	NASDAQ	1215700	SBUX	Starbucks Corp.	NASDAQ	1287200	0.80	0.45
	APOL	Apollo Group	NASDAQ	332200	SHLD	Sears Holdings Corporation	NASDAQ	317900	0.28	0.57
	AMZN	Amazon Corp.	NASDAQ	1215700	SPLS	Staples Inc.	NASDAQ	674500	0.80	0.62
	AMZN	Amazon Corp.	NASDAQ	1215700	CMCSA	Comcast Corp.	NASDAQ	910000	0.69	0.64
	EXPE	Expedia Inc.	NASDAQ	883300	SHLD	Sears Holdings Corporation	NASDAQ	317900	0.43	0.66
Consumer Staples	COST	Costco Co.	NASDAQ	237500	PG	Procter & Gamble	NYSE	1396077700	0.05	0.71
	COST	Costco Co.	NASDAQ	237500	CVS	CVS Caremark Corp.	NYSE	1454768200	0.09	0.77
	KO	Coca Cola Co.	NYSE	1152559200	COST	Costco Co.	NASDAQ	237500	0.04	0.79
	MO	Altria Group Inc.	NYSE	1312801100	CCE	Coca-Cola Enterprises	NYSE	335449400	0.29	0.79
	CCE	Coca-Cola Enterprises	NYSE	335449400	KFT	Kraft Foods Inc.-A	NYSE	1438368700	0.32	0.80
Energy	EP	El Paso Corp.	NYSE	697103700	SE	Spectra Energy Corp.	NYSE	331523100	0.34	0.84
	CVX	Chevron Corp.	NYSE	1271849400	SE	Spectra Energy Corp.	NYSE	331523100	0.26	0.85
	HES	Hess Corporation	NYSE	427802700	SE	Spectra Energy Corp.	NYSE	331523100	0.23	0.85
	MUR	Murphy Oil	NYSE	223916300	SE	Spectra Energy Corp.	NYSE	331523100	0.25	0.86
	SI	Smith International	NYSE	370893400	SE	Spectra Energy Corp.	NYSE	331523100	0.32	0.86
Financials	SCHW	Charles Schwab	NASDAQ	1445500	ETFC	E*Trade Financial Corp.	NASDAQ	1391800	0.67	0.44
	ETFC	E*Trade Financial Corp.	NASDAQ	1391800	FITB	Fifth Third Bancorp	NASDAQ	218000	0.29	0.58
	SCHW	Charles Schwab	NASDAQ	1445500	FITB	Fifth Third Bancorp	NASDAQ	218000	0.36	0.58
	SCHW	Charles Schwab	NASDAQ	1445500	HCBK	Hudson City Bancorp	NASDAQ	686900	0.54	0.60
	ACAS	American Capital Strategies Ltd	NASDAQ	207200	SCHW	Charles Schwab	NASDAQ	1445500	0.50	0.60
Health Care	CELG	Celgene Corp.	NASDAQ	619200	ESRX	Express Scripts	NASDAQ	998400	0.65	0.47
	AMGN	Amgen	NASDAQ	813900	CELG	Celgene Corp.	NASDAQ	619200	0.48	0.50
	AMGN	Amgen	NASDAQ	813900	BIIB	BIOGEN IDEC Inc.	NASDAQ	381800	0.48	0.52
	CELG	Celgene Corp.	NASDAQ	619200	THC	Tenet Healthcare Corp.	NYSE	805228900	-0.23	0.53
	AMGN	Amgen	NASDAQ	813900	GENZ	Genzyme Corp.	NASDAQ	242900	0.57	0.53
Industrials	GE	General Electric	NYSE	4303823300	LUV	Southwest Airlines	NYSE	862775700	0.53	0.73
	MMM	3M Company	NYSE	549124400	CBE	Cooper Industries Ltd.	NYSE	175911300	0.24	0.73
	CBE	Cooper Industries Ltd.	NYSE	175911300	GWV	Grainger (W.W.) Inc.	NYSE	95324400	0.15	0.73
	CBE	Cooper Industries Ltd.	NYSE	175911300	GR	Goodrich Corporation	NYSE	144177800	0.15	0.75
	CBE	Cooper Industries Ltd.	NYSE	175911300	FLR	Fluor Corp. (New)	NYSE	171713100	0.10	0.76
Information Technology	AAPL	Apple Inc.	NASDAQ	4627500	INTC	Intel Corp.	NASDAQ	5529100	0.84	0.19
	AAPL	Apple Inc.	NASDAQ	4627500	CSCO	Cisco Systems	NASDAQ	4886800	0.76	0.24
	AAPL	Apple Inc.	NASDAQ	4627500	YHOO	Yahoo Inc.	NASDAQ	2609700	0.71	0.25
	AAPL	Apple Inc.	NASDAQ	4627500	ORCL	Oracle Corp.	NASDAQ	1731900	0.78	0.25
	AAPL	Apple Inc.	NASDAQ	4627500	EBAY	eBay Inc.	NASDAQ	1056100	0.73	0.26
Materials	MON	Monsanto Co.	NYSE	479605700	SEE	Sealed Air Corp.(New)	NYSE	126716200	0.06	0.52
	FCX	Freeport-McMoran Cp & Gld	NYSE	1058215000	SIAL	Sigma-Aldrich	NASDAQ	133800	0.05	0.54
	ECL	Ecolab Inc.	NYSE	163404500	SEE	Sealed Air Corp.(New)	NYSE	126716200	0.26	0.60
	ATI	Allegheny Technologies Inc	NYSE	269746100	SEE	Sealed Air Corp.(New)	NYSE	126716200	0.10	0.63
	PX	Praxair Inc.	NYSE	245761900	SEE	Sealed Air Corp.(New)	NYSE	126716200	0.12	0.65
Telecommunication Services	Q	Qwest Communications Int	NYSE	1623807700	S	Sprint Nextel Corp.	NYSE	2044634000	0.52	0.84
	Q	Qwest Communications Int	NYSE	1623807700	VZ	Verizon Communications	NYSE	1472335800	0.49	0.86
	S	Sprint Nextel Corp.	NYSE	2044634000	VZ	Verizon Communications	NYSE	1472335800	0.49	0.87
	AMT	American Tower Corp.	NYSE	387199400	Q	Qwest Communications Int	NYSE	1623807700	0.26	0.97
	AMT	American Tower Corp.	NYSE	387199400	WIN	Windstream Corporation	NYSE	400634200	0.10	0.99
Utilities	DUK	Duke Energy	NYSE	902519200	DYN	Dynegy Inc.	NYSE	702035600	0.54	0.74
	CMS	CMS Energy	NYSE	264225200	DYN	Dynegy Inc.	NYSE	702035600	0.48	0.79
	CMS	CMS Energy	NYSE	264225200	DUK	Duke Energy	NYSE	902519200	0.39	0.81
	AES	AES Corp.	NYSE	556049300	CMS	CMS Energy	NYSE	264225200	0.31	0.84
	CNP	CenterPoint Energy	NYSE	359757800	DUK	Duke Energy	NYSE	902519200	0.33	0.84

Table B.2: Top 5 five stock pairs with the highest correlation from each GICS branch of the S&P 500 index.

GICS Branch	Stock 1				Stock 2				corr	varcorr
	Symbol	Name	Stock Exchange	Volume	Symbol	Name	Stock Exchange	Volume		
Consumer Discretionary	APOL	Apollo Group	NASDAQ	332200	SPLS	Staples Inc.	NASDAQ	674500	0.77	1.08
	BBBY	Bed Bath & Beyond	NASDAQ	233000	SPLS	Staples Inc.	NASDAQ	674500	0.79	1.08
	AMZN	Amazon Corp.	NASDAQ	1215700	SPLS	Staples Inc.	NASDAQ	674500	0.80	0.62
	AMZN	Amazon Corp.	NASDAQ	1215700	SBUX	Starbucks Corp.	NASDAQ	1287200	0.80	0.45
	SPLS	Staples Inc.	NASDAQ	674500	SBUX	Starbucks Corp.	NASDAQ	1287200	0.81	0.69
Consumer Staples	KO	Coca Cola Co.	NYSE	1152559200	SLE	Sara Lee Corp.	NYSE	542934700	0.42	0.90
	SLE	Sara Lee Corp.	NYSE	542934700	WMT	Wal-Mart Stores	NYSE	1992433400	0.43	1.02
	CVS	CVS Caremark Corp.	NYSE	1454768200	KFT	Kraft Foods Inc-A	NYSE	1438368700	0.44	0.98
	KFT	Kraft Foods Inc-A	NYSE	1438368700	SLE	Sara Lee Corp.	NYSE	542934700	0.48	0.87
	COST	Costco Co.	NASDAQ	237500	WFM	Whole Foods Market	NASDAQ	211400	0.59	1.09
Energy	EP	El Paso Corp.	NYSE	697103700	RDC	Rowan Cos.	NYSE	369346200	0.39	0.91
	CVX	Chevron Corp.	NYSE	1271849400	XOM	Exxon Mobil Corp.	NYSE	2798325600	0.41	1.00
	XOM	Exxon Mobil Corp.	NYSE	2798325600	HAL	Halliburton Co.	NYSE	1701703200	0.45	1.18
	CVX	Chevron Corp.	NYSE	1271849400	EP	El Paso Corp.	NYSE	697103700	0.45	0.88
	COP	ConocoPhillips	NYSE	1382115600	EP	El Paso Corp.	NYSE	697103700	0.46	0.94
Financials	CINF	Cincinnati Financial	NASDAQ	55900	TROW	T. Rowe Price Group	NASDAQ	216400	0.60	1.74
	FITB	Fifth Third Bancorp	NASDAQ	218000	HBAN	Huntington Bancshares	NASDAQ	71600	0.62	2.01
	FITB	Fifth Third Bancorp	NASDAQ	218000	TROW	T. Rowe Price Group	NASDAQ	216400	0.63	1.34
	SCHW	Charles Schwab	NASDAQ	1445500	ETFC	E*Trade Financial Corp.	NASDAQ	1391800	0.67	0.44
	TROW	T. Rowe Price Group	NASDAQ	216400	ZION	Zions Bancorp	NASDAQ	48500	0.70	1.40
Health Care	BIIB	BIOGEN IDEC Inc.	NASDAQ	381800	PDCO	Patterson Cos. Inc.	NASDAQ	106400	0.60	1.15
	BIIB	BIOGEN IDEC Inc.	NASDAQ	381800	GENZ	Genzyme Corp.	NASDAQ	242900	0.62	0.85
	GENZ	Genzyme Corp.	NASDAQ	242900	GILD	Gilead Sciences	NASDAQ	275100	0.64	0.91
	CELG	Celgene Corp.	NASDAQ	619200	ESRX	Express Scripts	NASDAQ	998400	0.65	0.47
	BSX	Boston Scientific	NYSE	1205569800	THC	Tenet Healthcare Corp.	NYSE	805228900	0.68	0.60
Industrials	GE	General Electric	NYSE	4303823300	LUV	Southwest Airlines	NYSE	862775700	0.53	0.73
	CTAS	Cintas Corporation	NASDAQ	48500	EXPD	Expeditors Int'l	NASDAQ	215600	0.54	1.48
	EXPD	Expeditors Int'l	NASDAQ	215600	MNST	Monster Worldwide	NASDAQ	196700	0.55	1.20
	CHRW	C.H. Robinson Worldwide	NASDAQ	112400	EXPD	Expeditors Int'l	NASDAQ	215600	0.56	1.46
	MNST	Monster Worldwide	NASDAQ	196700	PCAR	PACCAR Inc.	NASDAQ	164400	0.56	1.23
Information Technology	DELL	Dell Inc.	NASDAQ	909400	ORCL	Oracle Corp.	NASDAQ	1731900	0.79	0.47
	CSO	Cisco Systems	NASDAQ	4886800	DELL	Dell Inc.	NASDAQ	909400	0.79	0.41
	INTC	Intel Corp.	NASDAQ	5529100	ORCL	Oracle Corp.	NASDAQ	1731900	0.82	0.29
	CSO	Cisco Systems	NASDAQ	4886800	ORCL	Oracle Corp.	NASDAQ	1731900	0.83	0.34
	AAPL	Apple Inc.	NASDAQ	4627500	INTC	Intel Corp.	NASDAQ	5529100	0.84	0.19
Materials	DD	Du Pont (E.I.)	NYSE	716944300	FCX	Freeport-McMoran Cp & Gld	NYSE	1058215000	0.36	1.02
	FCX	Freeport-McMoran Cp & Gld	NYSE	1058215000	MON	Monsanto Co.	NYSE	479605700	0.36	0.85
	APD	Air Products & Chemicals	NYSE	187953500	BLL	Ball Corp.	NYSE	119560600	0.36	0.88
	ECL	Ecolab Inc.	NYSE	163404500	NEM	Newmont Mining Corp. (Hldg. Co.)	NYSE	958900000	0.37	0.93
	FCX	Freeport-McMoran Cp & Gld	NYSE	1058215000	NEM	Newmont Mining Corp. (Hldg. Co.)	NYSE	958900000	0.43	1.06
Telecommunication Services	T	AT&T Inc.	NYSE	2663617200	Q	Qwest Communications Int	NYSE	1623807700	0.45	1.01
	S	Sprint Nextel Corp.	NYSE	2044634000	VZ	Verizon Communications	NYSE	1472335800	0.49	0.87
	Q	Qwest Communications Int	NYSE	1623807700	VZ	Verizon Communications	NYSE	1472335800	0.49	0.86
	T	AT&T Inc.	NYSE	2663617200	VZ	Verizon Communications	NYSE	1472335800	0.50	1.05
	Q	Qwest Communications Int	NYSE	1623807700	S	Sprint Nextel Corp.	NYSE	2044634000	0.52	0.84
Utilities	DUK	Duke Energy	NYSE	902519200	TE	TECO Energy	NYSE	177983100	0.35	0.87
	D	Dominion Resources	NYSE	321656100	XEL	Xcel Energy Inc	NYSE	337262500	0.36	0.92
	CMS	CMS Energy	NYSE	264225200	DUK	Duke Energy	NYSE	902519200	0.39	0.81
	CMS	CMS Energy	NYSE	264225200	DYN	Dynegy Inc.	NYSE	702035600	0.48	0.79
	DUK	Duke Energy	NYSE	902519200	DYN	Dynegy Inc.	NYSE	702035600	0.54	0.74

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